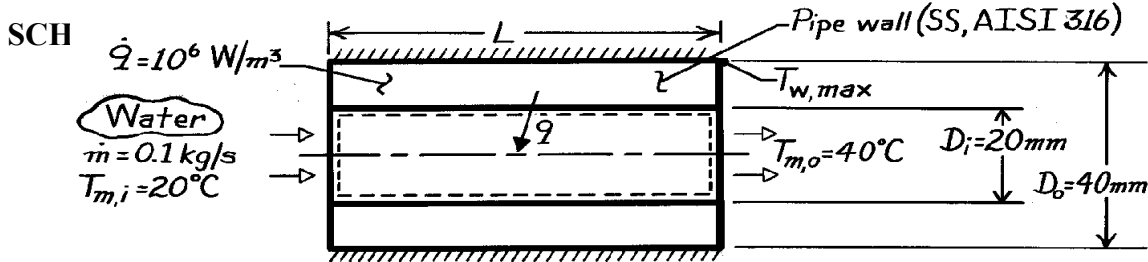


PROBLEM 06

KNOWN: Inner and outer diameter of a steel pipe insulated on the outside and experiencing uniform heat generation. Flow rate and inlet temperature of water flowing through the pipe.

FIND: (a) Pipe length required to achieve desired outlet temperature, (b) Location and value of maximum pipe temperature.



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible liquid with negligible viscous dissipation, (4) One-dimensional radial conduction in pipe wall, (5) Outer surface is adiabatic.

PROPERTIES: Table A-1, Stainless steel 316 ($T \approx 400\text{K}$): $k = 15\text{ W/m}\cdot\text{K}$; Table A-6, Water ($\bar{T}_m = 303\text{K}$): $c_p = 4178\text{ J/kg}\cdot\text{K}$, $k = 0.617\text{ W/m}\cdot\text{K}$, $\mu = 803 \times 10^{-6}\text{ N}\cdot\text{s/m}^2$, $\text{Pr} = 5.45$.

ANALYSIS: (a) Performing an energy balance for a control volume about the inner tube, it follows that

$$\dot{m} c_p (T_{m,o} - T_{m,i}) = q = \dot{q} (\pi/4) (D_o^2 - D_i^2) L$$

$$L = \frac{\dot{m} c_p (T_{m,o} - T_{m,i})}{\dot{q} (\pi/4) (D_o^2 - D_i^2)} = \frac{(0.1\text{ kg/s}) 4178 (\text{J/kg}\cdot\text{K}) 20^\circ\text{C}}{10^6\text{ W/m}^3 (\pi/4) [(0.04\text{m})^2 - (0.02\text{m})^2]}$$

$$L = 8.87\text{m.}$$

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(b) The maximum wall temperature exists at the pipe exit ($x = L$) and the insulated surface ($r = r_o$). From Eq. 3.56, the radial temperature distribution in the wall is of the form

$$T(r) = -\frac{\dot{q}}{4k} r^2 + C_1 \ln r + C_2.$$

Considering the boundary conditions;

$$r = r_o : \left. \frac{dT}{dr} \right|_{r=r_o} = 0 = -\frac{\dot{q}}{2k} r_o + \frac{C_1}{r_o} \quad C_1 = \frac{\dot{q} r_o^2}{2k}$$

Continued ...

PROBLEM 8.32 (Cont.)

$$r = r_i : \quad T(r_i) = T_s = -\frac{\dot{q}}{4k} r_i^2 + \frac{\dot{q} r_o^2}{2k} \ln r_i + C_2 \quad C_2 = \frac{\dot{q}}{4k} r_i^2 - \frac{\dot{q} r_o^2}{2k} \ln r_i + T_s.$$

The temperature distribution and the maximum wall temperature ($r = r_o$) are

$$T(r) = -\frac{\dot{q}}{4k} (r^2 - r_i^2) + \frac{\dot{q} r_o^2}{2k} \ln \frac{r}{r_i} + T_s$$

$$T_{w,\max} = T(r_o) = -\frac{\dot{q}}{4k} (r_o^2 - r_i^2) + \frac{\dot{q} r_o^2}{2k} \ln \frac{r_o}{r_i} + T_s$$

where T_s , the inner surface temperature of the wall at the exit, follows from

$$q_s'' = \frac{\dot{q}(\pi/4) (D_o^2 - D_i^2)L}{\pi D_i L} = \frac{\dot{q}(D_o^2 - D_i^2)}{4 D_i} = h(T_s - T_{m,o})$$

where h is the local convection coefficient at the exit. With

$$\text{Re}_D = \frac{4 \dot{m}}{\pi D_i \mu} = \frac{4 \times 0.1 \text{ kg/s}}{\pi (0.02 \text{ m}) 803 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 7928$$

the flow is turbulent and, with $(L/D_i) = (8.87 \text{ m}/0.02 \text{ m}) = 444 \gg (x_{fd}/D) \approx 10$, it is also fully developed. Hence, from the Gnielinski correlation, Eq. 8.62,

$$h = \frac{k}{D_i} \left[\frac{(f/8)(\text{Re}_D - 1000) \text{Pr}}{1 + 12.7(f/8)^{1/2}(\text{Pr}^{2/3} - 1)} \right]$$

$$= \frac{0.617 \text{ W/m} \cdot \text{K}}{0.02 \text{ m}} \left[\frac{(0.033618)(7928 - 1000)5.45}{1 + 12.7(0.033618)^{1/2}(5.45^{2/3} - 1)} \right] = 1796 \text{ W/m}^2 \cdot \text{K}$$

where from Eq. 8.21, $f = (0.790 \ln \text{Re}_D - 1.64)^{-2} = 0.0336$. Hence, the inner surface temperature of the wall at the exit is

$$T_s = \frac{\dot{q}(D_o^2 - D_i^2)}{4 h D_i} + T_{m,o} = \frac{10^6 \text{ W/m}^3 \left[(0.04 \text{ m})^2 - (0.02 \text{ m})^2 \right]}{4 \times 1796 \text{ W/m}^2 \cdot \text{K} (0.02 \text{ m})} + 40^\circ \text{C} = 48.4^\circ \text{C}$$

$$\text{and} \quad T_{w,\max} = -\frac{10^6 \text{ W/m}^3}{4 \times 15 \text{ W/m} \cdot \text{K}} \left[(0.02 \text{ m})^2 - (0.01 \text{ m})^2 \right]$$

$$+ \frac{10^6 \text{ W/m}^3 (0.02 \text{ m})^2}{2 \times 15 \text{ W/m} \cdot \text{K}} \ln \frac{0.02}{0.01} + 48.4^\circ \text{C} = 52.6^\circ \text{C}.$$

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COMMENTS: The physical situation corresponds to a uniform surface heat flux, and T_m increases linearly with x . In the fully developed region, T_s also increases linearly with x .

