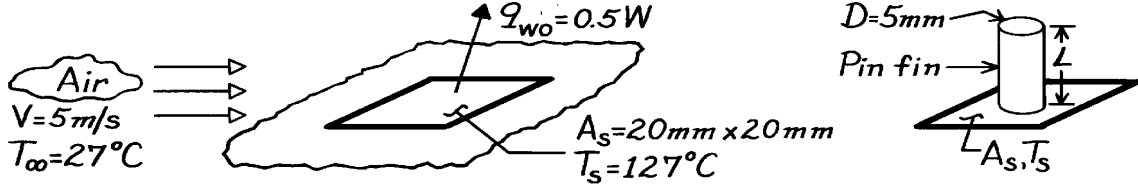


PROBLEM 02

KNOWN: Pin fin installed on a surface with prescribed heat rate and temperature.

FIND: (a) Maximum heat removal rate possible, (b) Length of the fin, (c) Effectiveness, ε_f , (d) Percentage increase in heat rate from surface due to fin.

SC



ASSUMPTIONS: (1) Steady-state conditions, (2) Conditions over A_s are uniform for both situations, (3) Conditions over fin length are uniform, (4) Flow over pin fin approximates cross-flow.

PROPERTIES: Table A-4, Air ($T_f = (T_\infty + T_s)/2 = (27 + 127)^\circ\text{C}/2 = 350\text{ K}$): $\nu = 20.92 \times 10^{-6}\text{ m}^2/\text{s}$, $k = 30.0 \times 10^{-3}\text{ W/m}\cdot\text{K}$, $\text{Pr} = 0.700$. Table A-1, SS AISI304 ($\bar{T} = T_f = 350\text{ K}$): $k = 15.8\text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) Maximum heat rate from fin occurs when fin is infinitely long,

$$q_f = M = (\bar{h}PkA_c)^{1/2} \theta_b \quad (1)$$

from Eq. 3.85. Estimate convection heat transfer coefficient for cross-flow over cylinder,

$$\text{Re}_D = \frac{VD}{\nu} = 5\text{ m/s} \times 0.005\text{ m} / 20.92 \times 10^{-6}\text{ m}^2/\text{s} = 1195.$$

Using the Hilpert correlation, Eq. 7.52, with Table 7.2, find

$$\bar{h} = \frac{k}{D} \text{CRe}_D^m \text{Pr}^n = (0.030\text{ W/m}\cdot\text{K} / 0.005\text{ m}) 0.683 (1195)^{0.466} (0.700)^{1/3} = 98.9\text{ W/m}^2\cdot\text{K}$$

From Eq. (1), with $P = \pi D$, $A_c = \pi D^2/4$, and $\theta_b = T_s - T_\infty$, find

$$q_f = \left(98.9\text{ W/m}^2\cdot\text{K} \times \pi (0.005\text{ m}) \times 15.8\text{ W/m}\cdot\text{K} \times \pi (0.005\text{ m})^2 / 4 \right)^{1/2} (127 - 27)\text{ K} = 2.20\text{ W}. <$$

(b) From Example 3.9, $L \approx L_\infty = 2.65(kA_c/hP)^{1/2}$. Hence,

$$L \approx L_\infty = 2.65 \left[15.8\text{ W/m}\cdot\text{K} \times \pi (0.005\text{ m})^2 / 4 / 98.9\text{ W/m}^2\cdot\text{K} \times \pi (0.005\text{ m}) \right]^{1/2} = 37.4\text{ mm}. <$$

(c) From Eq. 3.86, with h_s used for the base area A_s , the effectiveness is

$$\varepsilon_f = \frac{q_f}{h_s A_{c,b} \theta_b} = \frac{q_f}{q_{wo}} \frac{A_s}{A_{c,b}} = \frac{2.2\text{ W}}{0.5\text{ W}} \cdot \frac{(0.020 \times 0.020)\text{ m}^2}{\pi (0.005\text{ m})^2 / 4} = 89.6 <$$

where $h_s = q_{wo} / A_s \theta_b$.

(d) The percentage increase in heat rate with the installed fin (w) is

$$\frac{q_w - q_{wo}}{q_{wo}} \times 100 = \left(\left[q_f + h_s \left(A_s - \pi D^2 / 4 \right) (T_s - T_\infty) \right] - q_{wo} \right) \times 100 / q_{wo}$$

$$\Delta q/q = \left\{ \left[2.2\text{ W} + 12.5\text{ W/m}^2\cdot\text{K} \left([0.02\text{ m}]^2 - (\pi/4)(0.005\text{ m})^2 \right) 100\text{ K} - 0.5\text{ W} \right] \right\} \times 100 / 0.5\text{ W}$$

$$\Delta q/q = 435\%. <$$